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Mathematics News Letter

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To mathematics in general, to the following causes in particular is this journal dedicated: (1) the common problems of high school and college mathematics teaching, (2) the disciplines of mathematics, (3) the promotion of M. A. of A. and N. C. of T. of M. projects.

Editors: { S. T. SANDERS, Baton Rouge, La.
HENRY SCHROEDER, Ruston, La.

Louisiana-Mississippi Section, Mathematical Association of America

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ON THE NATURE OF MATHEMATICAL REASONING

The mind is deficient which has not some power to make inferences. This is the main business of the judge, the lawyer, the economist, the publicist, the mathematician and others. Scores of occupations in themselves require little or no use of an inferential process. For those engaged in such occupations, training in such a process, if had at all, must be had in the recreations and the schools.

In a fundamental sense the nature of an inferential process is the same in law as in morals, economics, or mathematics, or in any field that seriously employs reasoning. But it is in mathematics alone that the process exists in a state of perfection. Indeed one might define mathematics to be that science which makes use only of perfected inferential processes. A mathematical inference has two characteristics: (1), certainty, (2), accuracy. Hence to set forth sharply the difference between a mathematical and a non-mathematical inference it is sufficient to determine the elements which are necessary to a deductive process in order to make its results, (1), certain, (2), accurate.

Consider the following typical mathematical inference: If ¾ of a gallon of syrup is worth 90 cents, ½ of a gallon is

worth 60 cents. Resolved into a series of inferences the proposition may be stated thus.

If	¾	of a gallon	costs	90	cents,	(1)
then	¼	"	"	30	"	(2)
whence	1		"	120	"	(3)
whence	½	"	"	60	"	(4)

Only the ignorant or defective-minded person would deny (a), the accuracy, (b), the certainty of statement (4). Simple as the process actually is, a thorough study of its character must disclose the elements present which furnish these two characteristics.

We catalogue such elements as follows:

(a) The **meanings** of the terms (symbols) used, namely 3, 4, 1, 2, 90, 30, 120, 60, cent, gallon, are identical for all minds.

(b) The **meanings** of the **operations** associated with the symbols ½, ¾, ¼, are identical for all minds.

(c) The **certainty** that (2) follows from (1), (3) from (2), (4) from (3) arises from a mere **perceptive** act of the mind resulting from arithmetical analysis. Conscious perception is always **certain.**

The perception that (2) is a consequence of (1), etc., has the same element of certainty in it that belongs to an act of physically observing the finished product of a machine which has been made to operate on a given raw material in the presence of the observer.

(d) The meaning of each symbol employed, whether of number, of quantity, or of operation has a single, or unique value.

(e) The elements, or facts, of (1), the initial hypothesis, the material on which the inferential process operates, having the properties (a) and (b) described above, can actually be counted. In other words the hypothesis has no undetermined, or indeterminate, elements.

(f) The **accuracy** of (4), or, for that matter, of each inference, follows from (a), (b), (d).

(g) The initial "If", always present in every perfect logical process, signifies that the distinction between the absolute truth of an hypothesis and its conditional truth does not enter into the inferential process. In other words the final inference

is not to be affected by the truth or falsity of the statement that syrup is worth 120 cents a gallon.

(This analysis will be continued in the March issue of the News Letter)

—*S. T. S.*

MATHEMATICS AN AID

Laura Blank's article, "The Functions of Intuitive and Demonstrative Geometry," in the January Mathematics Teacher, should be read by every teacher of geometry and every defender of mathematics.

This article states that after failing in a suitable definition for the word demonstrate, Lincoln purchased the elements of Euclid.

One night Lincoln's friend found his companion learning to demonstrate propositions from geometry.

This is what was said of him. "He was trying to organize his mind and life so that he could not accuse himself, as he had accused President Polk, of being 'a bewildered, confounded, miserable, perplexed man.' He wanted to be simple as the alphabet, definite as the numbers used in arithmetic, sure as the axioms or common notions that are starting-points of Euclid. Had he trusted too much to his feelings, and not reasoned, proved, and demonstrated his propositions clearly enough in his own mind before speaking them during his term in Congress? He wasn't sure."

This is right in line with what was said in an address by Mr. Geo. B. Olds, President Emeritus of Amherst College, with reference to the brilliant English teacher that continued to take mathematics throughout his college course, because it helped him prune and clarify his English style.

—*H. S.*

NUMBERS

These few remarks about numbers are taken from the "History of Elementary Mathematics" by Cajori.

"Nearly all number-systems, both ancient and modern, are used on the scale of 5, 10, or 20. The reason for this is not

difficult to see. When a child learns to count, he makes use of his fingers and perhaps his toes. In the same way the savages of Prehistoric times unquestionably counted on their fingers and in some cases on their toes."

"Had the number of fingers and toes been different in man, then the prevalent number-systems of the world would have been different also."

"We are safe in saying that had one more finger sprouted from each human hand, making twelve fingers in all, then the numerical scale adopted by civilized nations would not be the decimal, but the duodecimal."

"As far as arithmetic is concerned it is certainly to be regretted that a sixth finger did not appear. The number 12 has for its exact divisors 2, 3, 4, 6, while 10 has only 2 and 5. In ordinary business affairs, the fractions 1/2, 1/3, and ¼ are used extensively, and it is very convenient to have a base which is an exact multiple of 2, 3, and 4."

"The Pythagoreans sought the origin of all things in numbers; harmony depended on musical proportions; the order and beauty of the universe have their origin in numbers; in the planetary motions they discerned a wonderful "harmony of the spheres." Moreover, some numbers have extraordinary attributes. Thus 1 is the essence of things; 4 is the most perfect number, corresponding to the human soul. According to Philolaus, 5 is the cause of color, 6 of cold, 7 of mind, health, and light, 8 of love and friendship. Even Plato and Aristotle refer the virtues to numbers. While these speculations in themselves were fantastic and barren, lines of fruitful mathematical inquiry were suggested by them."

"Isidorus (570-636), bishop of Seville, in Spain wrote an encycolopaedia, entitled Origines. He divides numbers into odd and even, speaks of perfect and excessive numbers, etc., and finally bursts out in admiration of numbers, as follows: Take away numbers from things, and every thing goes to destruction."

—*H. S.*

TO THE MEMBERS OF THE COUNCIL

The October issue of the Mathematics Teacher carried a notice to all members of the national council to this effect:

There are now 5,000 members and we have set ourselves the task of securing 10,000 members by 1930. To do this each member is asked to get a new member.

It is our aim to keep the Louisiana-Mississippi Branch well up among the states in number of members in the council. Therefore we earnestly solicit the cooperation of each member and urge him to secure at least one other.

If information regarding the work of the council is desired, it can be secured from Mathematics Teacher, 525 West 20th St. New York City. Circulars advertising the Mathematics Teacher and yearbooks will be sent to those who agree to use them in situations where they will do good.

Membership in the national council brings with it subscription to the Mathematics Teacher. Every phase of high school mathematics is discussed in this magazine. It would be difficult to try to estimate the value of this journal to the teacher of high school mathematics.

—*H. S.*

MATHEMATICS AND THE SEVEN AIMS OF SECONDARY EDUCATION

By MRS. LEOLA NIXON
Ruston High School

At first thought it might be concluded that mathematics contributes very little to the seven aims of secondary education. A closer study reveals that it furthers some of these aims very directly, and others in a marked degree, through its conceptual and transfer value.

Inglis tells us that with regard to the opportunity which mathematics affords for the exercise of those traits most desirable to transfer, it is equalled by few and surpassed by none of the other subjects taught in the secondary school.

All teachers, whether of mathematics or other subjects, should give some direct attention to the first objective, that of health. Though not the subject itself, the teacher of the class should given attention to the healthful surroundings of his pupils, to their posture, and to their general mental attitude toward health.

Secondary mathematics contributes strongly on the second point, the command of fundamental processes. The art of computation, while introduced and largely learned by rote in the elementary school, is not fully understood until studied and practised in the high schol. Computation should be progressively better understood, and more easily and frequently applied because of secondary school mathematics.

Mathematics has a very close relation, in some of its aspects, to certain vocations. Not in a broad sense, but in a very real way it has a specific vocational value. The vocational value of mathematics increases as the student goes beyond the high school, but the high school study is a necessary foundation for the advanced. Scientific experiment, inventions, research work in many lines, demand a knowledge of mathematics. An earnest successful physician in one of our northern cities, after he had reached middle life, went back to school and took up the mathematics that he had omitted in high school thinking that it would not concern his profession, because he found that he could not proceed with certain important lines of work to relieve mankind of disease, without a knowledge of this once neglected subject.

Where mathematics itself does not seem to contribute in the secondary school to health, worthy home membership, worthy use of leisure, citizenship, and ethical character, all of these are furthered and built up, in that person trained to logical reasoning; to an appreciation of the beauty and symmetry of the forms of nature and of art; to precision of thought; to a discrimination of the true and the false. The real lover of mathematics by his very manner and method of teaching this wonderful science upon which all nature rests, will cultivate and inculcate these very traits.

Some deny that mathematics has any power to build ethical character. I would refer such one to an article written several years ago by that prince of mathematicians, Dr. David Eugene Smith, entitled **Religio Mathematici** and published in the **Mathematics Teacher.** Geometry should especially contribute to this aim, for as Plato says "God eternally geometrizes." A distinctly religious sentiment may be aroused by creating in the student an appreciation of the vastness of space, of the power of mathematics, its wonderful accomplishments, its stability, its infinity.

METHODS OF PRESENTATION

PAULINE MOORE
Ruston High School

In the study of the pedagogy of mathematics, the point of view is sometimes that of the manner in which the subject matter is arranged and developed; other times, it is that of the manner in which it is presented to the pupils. This is covered by certain leading methods in mathematics such as, the inductive, deductive, the heuristic and project methods.

The deductive method proceeds from the general to the particular; the inductive from the particular to the general. For the learner the inductive method of approach is as a rule decidely the best. The teacher cannot study too carefully the roles that inductive and deductive reasoning play in mathematics, but it needs hardly be said that the pupil would profit little by any formal discussion of these methods. His attention should be confined to the actual reasoning. As to the work that pupil is asked to do, the opinion is widely held that inductive work should be given a more prominent part in the class-room work. Through deductive work the child does not develop a mode of thought but only acquires well established laws and facts.

The heuristic method is more commonly known as the development method and refers to that mode of teaching in which the child is led to rediscover the necessary knowledge that the race has gathered. Its aim is not the giving of facts, but the development of power through self-expression. An example of heuristic method would be: teaching children to subtract 37 from 62. Six dimes and two cents are shown. It is necessary to pay a debt of 37 cents, that is, of three dimes and seven cents. It is a simple matter to give my creditor three of the six dimes, but how give seven cents when I have only two? The children are now confronted by a problem. What solution can they suggest? A few more similar problems are solved and the children discover a way of subtracting numbers of two orders where the digit in the subtrahend is larger than the digit in the minuend. The heuristic method is therefore a mode of instruction which stirs self-activity and arouses a feeling of confidence in a child. Step by step he follows the line of thought, holds to it at every turn and finally sums up its substance.

The project method does not limit itself to the physical activities alone but makes provisions for acts of other types, provided that the individual takes a part in the purpose, choice and reflection of the directed action. There are different examples of projects which may be used, such as, the project of raising chickens. The child may be given a number of chickens. He is to have charge of these with the understanding that he is to familiarize himself with the best method of feeding and caring for them. He is furnished the chickens to start the project and is to do all the work that can be done by him when not attending school. The teacher may then cooperate by answering questions, furnishing literature and giving advice. In the lower grades a project may be made with a game of bean-bag, thus developing the aim of speed and accuracy in the addition of simple number combinations. If we can arouse the child's interest in the accomplishment of a purpose, we shall have the material for a project.

Sometimes teaching should be adapted to the individual because the needs of individuals differ. Every good teacher aims to reach his pupils individually, as far as possible. By observing and putting into practice these or similar methods, either of which may be suited to the class, will help to answer the important question that is asked by many teachers, how best to guide the pupil.

A BIT OF PERSONAL EXPERIENCE

By H. E. SLAUGHT
Professor of Mathematics, University of Chicago

I have never tried to act as a judge to decide on technical and theoretical grounds which group of the psychologists have the balance of evidence in their favor with respect to the "transfer of training" in the domain of mathematics. We shall probably never see that question finally closed. It very likely may be a question wherein faith intervenes when sight fails, and if so then, like all other matters of faith, it must rest upon individual experience. I venture, therefore, to occupy a little space in the News Letter to relate an incident in my experience where I firmly believe that faith was turned to sight.

When I was a student in an academy devoted entirely to preparation for college, I met my first mathematical thrill in beginning the study of algebra under a teacher who was not a mere lesson hearer but who put due emphasis upon the reasons for every step in a solution and insisted upon our seeing the logic involved in every process. That was my first introduction to precise thinking and I was deeply impressed with its significance and importance as compared with the flood of loose thinking and random guesses with which one is surrounded in the ordinary associations of every day life.

But this thrill from the study of algebra was as nothing compared to that which I was to receive when we began geometry. To set up a hypothesis, and to draw an inescapable conclusion therefrom—this was indeed a new game and it was full of thrills. I well remember how I went from one conquest to another not only with the theorems in the text but also with the "originals" which I laid low with especial glee. I soon found that my success depended upon several things: (1) A clear understanding of the hypothesis—it was even a first class exercise in clear and intelligible reading; (2) a definite comprehension of what I was trying to prove—an application of the scriptural injunction "He who goeth to warfare first sitteth down and counteth the cost"; and (3) an intelligent selection of the tools and methods at my disposal and an effective use of these in marching step by step from the hypothesis to the conclusion. And still another element of vast importance entered into the consideration of all theorems stated but not proved in the book; namely, the numerous chances of going wrong or of taking steps legitimate in themselves but leading to no effective results,—a situation quite analogous to the decisions that have to be made constantly in the ordinary affairs of life.

I pondered the significance of these things many a time and wondered whether I could transfer the habits of exact thinking, of logical argument, and of drawing conclusions based on reasoning, all of which I was gaining from my study of geometry, to other situations non-mathematical in character. It so happened that in this academy the teacher of geometry was also the teacher of English, or, more exactly, there was no department of English, but every instructor was responsible for the kind of English used by his students in both written and oral

exercises. Among other requirements was a frequent essay or theme from the members of this geometry class. For some time I drifted along, like most of my classmates, getting my themes here and there in haphazard fashion without rhyme or reason, when it suddenly occurred to me that my geometry should furnish the structural basis of my themes, that is, that a theme should have a background analogous to the hypothesis of a theorem, that it should undertake to establish some thesis or to reach some conclusion by means of an argument or a process of reasoning analogous to the proof of a theorem,—not necessarily deductive in character but certainly involving a consequential process of thought.

I put this procedure to the test and it produced a profound impression upon all my subsequent theme production. It put new life into theme writing and added a new zest to what had before been dull and without interest. Whatever may be the theoretical status of "transfer" I can testify that in my case transfer did take place and that whatever of clarity, force, and effectiveness may have characterized my processes of thinking and use of English during these forty years, I am certain that these results are due in large measure to the effect of my study of algebra and geometry in those preparatory days.

I can recommend to the young people of today the study of mathematics as an instrument for the development of precise thinking, of correct inference, and of clear and cogent expression, not only in the realm of mathematics itself but also in any realm where such attributes of the mind are in demand.

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*THE MINIMUM TRAINING PREREQUISITE TO THE SUCCESSFUL PURSUIT OF COLLEGE MATHEMATICAL STUDY

By A. C. MADDOX
Head Math. Dept., Louisiana Normal College

(Part Three)

The high school student should develop the ability to factor readily those types of algebraic expressions that are ordinarily used for that purpose in high school textbooks. He should de-

*Read before the Mathematics Section of L. T. A. Nov. 24, 1928.

velop the ability to simplify the types of radical expressions ordinarily found in the textbooks. He should become able to solve readily any ordinary quadratic equation in one unknown. It is particularly important that he master the development, the meaning, and the application of the quadratic formula. He should become able to solve any concrete problem of reasonable difficulty requiring the use of a quadratic equation in one unknown. He should develop the ability to solve graphically any system of first degree equations in two unknowns and to solve graphically any quadratic equation in one unknown.

It is of primary importance that students of algebra master the technic of forming equations that state in symbols the conditions that are stated in words in descriptions of problem situations.

The student's study of the laws of exponents should be very thoughtful rather than mechanical. He should be led to see that, so far as positive integral exponents are concerned, all the laws of exponents are immediate consequences of the agreement to represent $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$ by 5^6. In this connection it is quite necessary that the student understand that the meaning assigned to fractional exponents is such as will make the laws of fractional exponents identical with the laws of positive integral exponents. For example, the meaning of $8^{2/3}$ must harmonize with the fact that $8^{2/3} \cdot 8^{2/3} \cdot 8^{2/3} = :8^2$, i. e. $8^{2/3}$ must be $\sqrt[3]{8^2}$.

If a student is really to succeed with his work in geometry, he must acquire a thorough understanding of the different types of assumptions in geometry, i. e., of definitions, general axioms, geometric postulates, and problem hypotheses. He must become familiar with the general types of reasoning employed in elementary geometry and must develop the ability to choose the type that best serves any particular purpose. He must develop the ability to employ a correct and natural style of oral or written English in connection with geometric investigations and findings.

The high school student of geometry must become familiar with the theorems that are necessary in the sequences of theorems that constitute the subject matter generally accepted as basic in the Euclidean geometry of the plane. And the training afforded through the discovery and development of these the-

orems is one of the most necessary and fruitful types of mathematical training it is possible for a high school student to procure. And the degree of genuine success possible in college mathematical study is determined largely by the degree of real mastery acquired by the student in this part of high school geometry. This implies that the student should attack all theorems as "originals", so far as he can develop the power to do so effectively, and that he must develop the ability to solve, with reasonable facility, the various types of problem situations requiring the ordinary applications of the basic propositions.

It is particularly important that the high school student acquire a thorough understanding of the meaning, the proofs, and the applications of the elementary propositions involving loci, similarity of geometric figures, or the proportionality of line segments or of areas.

It is very important that the high school student develop some ability and inclination to formulate and set before himself problems for investigation and to pursue the investigations until he has solved the problems or until he has discovered that their solution requires some knowledge not yet discovered by himself in a logical sequence of discoveries. The training procured by consistent practice in this sort of thing will be of inestimable value to the student in his college mathematical study.

The high school student should acquire a thorough knowledge of, and adequate experience in, the various tests available in determining whether or not a given geometric figure satisfies the conditions stated or implied in a desired conclusion, as, for example, the various tests for the conguence of triangles or the different tests for parallelism or perpendicularity of straight lines.

The student must know which of the basic theorems have practical use only, or mainly, as links in a chain of theorems leading to some theorem that has numerous direct applications in computation or construction exercises and problems, it being particularly important that the mastery of such a final theorem be undoubtedly permanent. For example, the proposition expressing the area of any parallelogram in terms of its base and altitude is the final link in a chain of several theorems found in high school geometry and should, therefore, receive a type and

a degree of emphasis not so necessary in the case of the other theorems of the group.

The harmful effect upon students of merely memorizing the proofs of theorems for recitation and examination purposes can hardly be exaggerated. Accordingly, the mathematical training of high school students should be such as will discourage this practice as much as possible. A resourceful teacher can find numerous ways of directing the study of high school pupils in such manner as to hold up before them the goal of developing mathematical power rather than that of memorizing enough of the author's formal proofs to secure a unit of credit in the subject of geometry. And the teacher's method of checking up on the quality of work done by students may easily be such as will make it impossible for a student to earn his unit of credit merely by the use of his ability to memorize what somebody else has thought out.

There are some facts of geometry, however, that should be so well committed to memory, after they are developed and thoroughly understood by the student, that he will keep them as permanently as he keeps the fundamental multiplication combinations. And without the ability to apply such facts readily, the college student of mathematics is very seriously handicapped. I think I should not undertake to state here all the facts that I should include in such a list of facts. Quite a good many of the theorems whose proofs are given in the ordinary textbook in geometry should be included. The list should have in it, also, such facts as these: In a right triangle whose acute angles are 30° and 60° respectively the hypotenuse is double the shorter side. The diagonal of a square equals $\sqrt{2}$ times the side. A triangle is obtuse, right, or acute according as the square on the longest side is greater than, equal to, or less than the sum of the squares on the other two sides. Any triangle whose sides are in the ratios 3:4:5 or 5:12:13 is a right triangle.

LOUISIANA COLLEGE NEWS

By C. D. SMITH
Head of Mathematical Department

It has been suggested by the editor that an occasional word from the schools would be of interest to readers of the News

Letter. Our classes have been somewhat reorganized since the death of our beloved fellow teacher Prof. A. M. Hendon. He will be remembered by many as one of Louisiana's oldest and most faithful teachers. With Prof. C. W. Hunt dividing his time between freshman mathematics and biology and the writer taking the rest of the burden we have been able to command the situation to date. The course in geometry for freshmen has been discontinued with a view to offering later a 3-hour course in college geometry.

We now have 27 students electing advanced courses in mathematics. There are pressing demands for courses in reading, higher geometry, mathematical statistics and astronomy. For schools of liberal arts where the upper classes number about 200 it seems that such conditions should be taken as a hopeful sign. I believe if we could sell mathematics to our under-graduates a majority of them could profit by certain courses above the freshman work.

On the opening day of the fall term we had 86 freshman to report for the placement test in mathematics. A statistical study has been made of the results as compared with the term record of these students. Certain observations should be of interest to those who are interested in placement programs. The Iowa Placement Test M. T. I. revised was used as a basis for sectioning in three levels A, B, and C. We list the following observations:

1. The mean placement score was considerably below the normal.
2. The performance was also below normal.
3. All of section A passed the term's work.
4. All of section C failed except 2.
5. Section C was composed of high school graduates who made below 15% on the placement test.
6. We had 15 who took the placement test and failed to remain in the class until the close of the term.
7. We do not know the causes of the withdrawals but the records show that 12 of those who dropped were among the C's while not a single A dropped the class.
8. The correlation coefficient for placement scores and term averages is $r=.61$, while the normal is $r=.6$.

These observations are typical of what you will get with

any placement examination which gives r as high as .6. We have three options: first, require those who place below 15% to take a term of review work without college credit; second, lower standards and teach the kind of elementary mathematics they could pass; third, keep the standards where they are as regards the content of the first course and let them fail and repeat.

EDUCATIONAL THEORY REDUCED TO MATHEMATICAL FORM

By W. PAUL WEBBER
Louisiana State University

(Second Paper)

11. **Assumption.** There is an objective of education which must be found in the civilization (environment) in which those educated are to live.

12. **Assumption.** Elementary education should make the best possible provision for those who will not attain higher education.

Theorem IV. Subjects of instruction and training processes that most immediately promote an understanding of living in one's environment should receive first place in elementary education.

This follows from 8, 11, 12, Theorem 5.

13. **Assumption.** Communication of thought is necessary in understanding civilization.

Theorem V. Thorough knowledge of "Mother Tongue" is essential to all education.

This follows from Theorem IV and 13.

14. **Assumption.** Higher education should provide knowledge and training that prepare for control of, modification of, the environment for the general benefit and the dissemination of such knowledge.

15. **Assumption.** Education should prepare for the production of entertainment and pleasure for the general benefit.

16. **Assumption.** Understanding of the "fundamental laws" of nature, physical and human, is necessary to the carrying out of purposes (14) and (15).

17. **Historical Observation.** The laws of nature are exact and quantitative and require mathematical setting for their ap-

plication on a large scale suitable for the general benefit.

This is true in varying degree of all the different fields. The tendency of all is to greater mathematical development. All the civilized nations take part in these developments.

Theorem V. Knowledge of foreign languages is helpful in the life of the individual, especially the more highly educated.

This follows from 13, 16, 17.

Theorem VII. A modicum of fine arts should be included in education.

This follows from 15.

Theorem VIII. Knowledge of mathematics should be included for those who are to take part in the purposes of (14).

This follows from 14, 16, 17.

Remark. This would include scientific research, engineering of all kinds, economics, certain kinds of social welfare work, commerce, etc.

Theorem XI. Psychology should be included in all higher education.

This follows from 13, 14, 15, 16.

18. **Psychological Principle.** Generalization is the greatest function of the mind for promoting understanding of things and the application of knowledge to new situations.

19. **Assumption.** Knowledge of logical sequence of relations is necessary to correct thinking.

20. **Observation.** Thorough training in mathematical studies specifically trains in logical sequence and in generalization.

Theorem X. Mathematics can be profitably pursued by all normal minds.

This follows from 18, 19, 20.

Theorem XI. Knowledge of physics is necessary to all engineering.

This follows from 14, 16.

It is not to be supposed that the above theorems are all that can be derived from the principles and assumptions laid down. The number can be increased to large proportions. Further, it is not to be supposed that in this first effort that the best choice of principles and assumptions has been made or that the list is complete. The method of attack has been pointed out. The author hopes that at some future time he may carry this study further.

PERFECT NUMBERS

By ZENA GARRETT
Mississippi Delta State Teachers College

Perfect numbers—that is, those equal to the sum of their integral subdivisors—were, in very olden times, a matter of grave consideration and conjecture. The ancients explained many mysteries of Nature by the magic number 6, presumably the first of the perfect numbers, unless 1, or unity, be so considered.

By 500 B. C., we find the Pythagoreans classifying numbers as excessive, perfect, or defective, in comparison with the sum of their integral subdivisors. In the 3rd century B. C., Euclid discussed perfect numbers and concluded by giving a formula for their selection.

In the Middle Ages, theology adopted perfect numbers, and the theologians, led by Alcuin, explained many of the Bible facts by this symbolism. Because of the perfection of the number 6, God created the world in six days, and on the sixth day He created His most perfect work, man. God, who did all things well, created six beings in the first creation. The second origin of man emanated from the number 8, the number of souls said to have been in Noah's Ark. But 8 is a defective number, hence the imperfection of the second creation.

These numbers, though no longer robed with their mystic qualities, were still of interest to mathematicians of later ages. Marin Mersenne, a Frenchman of the 17th century, a Franciscan friar, published in 1464 the first eight perfect numbers. A ninth was discovered in 1885 and a tenth in 1912.

My interest in perfect numbers was first aroused by a discussion in class. Miss Dale explained to us the reason for the name, told us the ancient beliefs concerning them, and gave us the first two—6 and 28. Then she asked if we would be interested in finding the next one, which was between 28 and 500. We were instructed to work alone and forbidden the use of the library.

My first step was to examine closely the structure of the two numbers given us. They are both even numbers. The factors of 6 are 1, 2, and a prime, 3. The factors of 28 are 1, 2, (2^2), 7(a prime), and multiples of 7 and 2. Neither is divisi-

ble by any other number than a prime and powers of 2. I found that numbers divisible by multiples of 3 are excessive when even, defective when odd. Thus were all odd numbers and all numbers divisible by three eliminated as unlikely. Likewise the multiples of 5.

I soon found that perfect numbers did not appear regularly, that is, by the addition of or multiplication by a certain fixed amount; nor could I establish any satisfactory proportion. This made me think the number would probably be near 500, the limit given us.

With these conjectures as a basis, I soon found 496, the next perfect number after 28.

Now curiosity led me to discover the composition. The subdivisors of 496 are also based upon the powers of 2 and a prime—in this case, 31. The prime for 28 is 7, for 6, it is 3. In each case you will notice that it is a power of 2 minus 1. In 6, 2 is raised to the first power, in 28 to the second, in 496 to the fourth.

I could now write out the formula for 6, 28, and 496, respectively:

Let $x=\frac{1}{2}$ the perfect number N.

$\therefore 2x=N$

$1+2+x=2x$

$x=3$

N, or $2x=6$

$$1+2+x+(2^2)+\frac{x}{2}=2x$$

$2x=28$

$$1+2+x+4+\frac{x}{2}+8+\frac{x}{4}+16+\frac{x}{8}=2x$$

$x=248$

N, or $2x=496$

Now I tried my formula for the next one, thus:

$$1+2+x+4+\frac{x}{2}+8+\frac{x}{4}+16+\frac{x}{8}+32+\frac{x}{16}=2x$$

$x=4064$

N, or $2x=8128$

By testing I found this answer correct. I also found that the addition of one more set (e.g., $+32+\frac{x}{16}$) to the preceding ones, would not give a perfect number.

But this formula was cumbersome to work with, so I shortened it by the use of geometric progressions:

$$(1+2+4+8+16+\text{etc})+(x+\frac{x}{2}+\frac{x}{4}+\frac{x}{8}+\text{etc.})=2x=N$$

$$\left\{\frac{1-2l}{1-2}\right\}+\left\{\frac{x-\frac{1}{2}\left(\frac{2x}{l}\right)}{1-\frac{1}{2}}\right\}=2x$$

$$(2l-1)+\left\{\frac{2xl-2x}{l}\right\}=2x$$

$$2l^2-l+2xl-2x=2xl$$

$$2l^2-l=2x$$

$$l(2l-1)=2x=N$$

$$l=2^{n-1}$$

$$2^{n-1}(2^n-1)=N$$

2^n-1 must always be prime, as it is the prime upon which the perfect number is based. There seems to be no rule for ascertaining what values of n will make 2^n-1 prime, except that n will always be either 1 or an even number.

One of the peculiarities of perfect numbers is the fact that they always end in either 6, or 28. Another is the long, irregular leaps between the numbers.

But one thing to be regretted about perfect numbers is that nothing in Nature or the Universe has yet been discovered which has any association whatever with this vagary of our number system. Such an association is certainly highly improbable because of the immense rapidity with which the numbers increase. Still we wonder that they could be useless. It would be strange indeed if, in all the Universe, no other thing should follow the path of perfect numbers.

A TANGENCY PROBLEM

By R. L. O'QUINN
Louisiana State University

Problem. To find the circle through the points $P_1(x_1 y_1)$ and $P_2(x_2 y_2)$ tangent to the circle $x^2+y^2+2Ax+2By+C=0$. The required center lies on the perpendicular bisector of $P_1 P_2$. Parametric equations of the perpendicular bisector are:

$x=\frac{1}{2}(x_1+x)+\frac{1}{2}(y_1-y_2)t, y=\frac{1}{2}(y_1+y_2)-\frac{1}{2}(x_1-x_2)t$. Let (u,v) be a point in the plane. It lies on this perpendicular bisector if, and only if, $u=\frac{1}{2}(x_1+x_2)+\frac{1}{2}Yt_0$, $v=\frac{1}{2}(y_1+y_2)-\frac{1}{2}Xt_0$, where $X=x_1-x_2$, $Y=y_1-y_2$. Let

(1) $R_1=\sqrt{(u-x_2)^2+(v-y_2)^2}$, the distance from (uv) to P_2. The center of the given circle is $(-A,-B)$ and its radius is $\sqrt{A^2+B^2-C}$. Call the radius R_2. The distance from (uv) to $(-A,-B)$ is

(2) $\sqrt{(u+A)^2+(v+B)^2}$. Call this distance d_1. The circle whose center is (uv) and which passes through P_2 is tangent to given circle with center $(-A,-B)$ if, and only if

(3) $(R_1+R_2-d_1)(R_1-R_2-d_1)(R_1-R_2+d_1)(R_1+R_2+d_1)=0$, or
$[R_1+d_1)^2-R_2^2][(R_1-d_1)^2-R_2^2]+=0$.

From (3) is obtained

(4) $[(R_1^2-R_2^2+d_1^2)-2d_1R_1][(R_1^2-R_2^2+d_1^2)+2d_1R_1]=0$, or
$(R_1^2-R_2^2+d_1^2)^2-4d_1R_1=0$.

Now, the polar of the circle $x^2+y^2+2Ax+2By+C=0$ with respect to the point P_1 is $x_1x+y_1y+A(x+x_1)+B(y+y_1)+C=0$. The value of the expression on the left of the equality sign, upon substituting the coordinates of P_2 for x and y, is

(5) $x_1x_2+y_1y_2+A(x_1+x_2)+B(y_1+y_2)+C$. Call this P. The area of the triangle determined by P_1, P_2 and $(-A,-B)$ is

(6) $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ -A & -B & 1 \end{vmatrix}=\frac{1}{2}[x_1y_2-Bx_2-Ay_1+Ay_2-x_2y_1+Bx_1]$. Call this area k, and set $s=\overline{P_1P_2}$. Then, in equation (4),

$R_1^2=\frac{1}{4}(s^2+s^2t_0^2)$, $R_2^2=A^2+B^2-C$, and

$d_1^2=\frac{1}{4}(s^2t_0^2-8kt_0+4P+s^2+4R^2)$. Hence

$R_1^2-R_2^2+d_1^2=\frac{1}{2}(s^2t_0^2-4kt_0+2P+s^2)$, and

$d_1^2R_1^2=1/16(s^2+s^2t_0^2)(s^2t_0^2-8kt_0+4P+s^2+4R^2)$. Substituting these values in equation (4) we get the equation in t_0:

(7) $(s^2R^2-4k^2)t_0^2+4kPt_0+(s^2R^2-P^2)=0.$

Case I. Suppose $s^2R^2-4k^2=0$. This means that $\overline{P_1P_2}^2R^2=4k^2$. Let h be the altitude of the triangle P_1P_2O, where O is the fixed center (—A,—B). Then $\overline{P_1P_2}^2\cdot R^2=4\left\{\frac{h\cdot P_1P_2}{2}\right\}^2$, or

$|h|=R$. The geometric meaning is that P_1P_2 is tangent to the circle with center 0. The equation (7) reduces to the first degree, and there are two possible cases: (a) If P differ from 0, then kP is different from 0, and $t_0=\frac{P^2-aR^2}{4kP}$. If $P^2-aR^2=0$, then $t_0=0$ and (u,v) is the mid-point of P_1P_2. (b) If $P=0$, then (7) reduces to $s^2R^2=0$, which is false. For P to be 0 means geometrically that one of the given points is on the polar of the other with respect to the given circle. K is never zero, because P_1 and P_2 are distinct and 0 is not on the line P_1P_2.

Case II. If $s^2R^2-4k^2$ differ from 0, then equation (7) is a quadratic in t_0 whose solution is given by

(8) $t_0=\frac{-2kP+Rs\sqrt{T_1T_2}}{s^2R^2-4k^2}$, or $t_0=\frac{-2kP-Rs\sqrt{T_1T_2}}{s^2R^2-4k^2}$

where T_1 and T_2 are

$x_1^2+y_1^2+2Ax_1+2By_1+C$ and $x_2^2+y_2^2+2By_2+C$, respectively.

Hence the problem has two solutions, one solution, or no solution according as T_1T_2 is $\gtreqless 0$; that is, acording as P_1 and P_2 are both outside the circle or both inside, or at least one of the given points on the circle, or one point inside and the other outside the circle.

NOTE ON PARAMETIC EQUATIONS OF CERTAIN DEGENERATE CONICS

By H. L. SMITH
Louisiana State University

If the reader were asked to factor the expression

(1) $ax^2+2bxy+cy^2$,

he would probably give two different forms, one for a different from zero and one for a equal to zero. In problems involving

literal quantities such arbitrary subdivision into cases is often inconvenient, and it would be desirable to have a factorization of (1) which would be valid without any restrictions on the coefficients. The writer does not know of such a factorization which is simple and natural. But it is the purpose of this note to show that the degenerate conic

(2) $$ax^2+2bxy+cy^2=0$$

has parametric equations which are valid without any restrictions whatever on the coefficients. This result may be stated in precise form as follows:

The numbers x, y will satisfy (2) if, and only if, there is a number t such that

(3) $$x=(r-b)t,\ y=at$$

or such that

(4) $$x=ct,\ y=(r-b)t,$$

where r is either square root of b^2-ac if ac is not zero, but is $-b$ if ac is zero.

We note first that the relation

(5) $$r^2=b^2-ac$$

holds whether or not ac is zero. Now set

$$F_1=ax-(r-b)y,\ F_2=(r-b)x-cy.$$

Then

$$F_1F_2=a(r-b)x^2-[ac+(r-b)^2]xy+c(r-b)y^2.$$

But by (5)

$$ac+(r-b)^2=b^2-r^2+(r-b)^2=-2b(r-b).$$

Hence

$$F_1F_2=(r-b)(ax^2+2bxy+cy^2).$$

If now $r-b$ is not zero, then (2) will hold if, and only if, $F_1=0$ or $F_2=0$, and hence if, and only if, there is a t such that (3) holds or such that (4) holds.

It remains to consider the case in which $r-b$ is zero. Here by (5) $ac=0$ and therefore $b=0$. Hence (2) reduces to

$$ax^2=0 \quad \text{or} \quad cy^2=0$$

according as a or c is different from zero. The truth of the theorem is then obvious.

THE LAFAYETTE MEETING

According to a recent telegraphic communication from Chairman Mitchell the joint meeting of the La.-Miss. Section and Council has been fixed for March 29 and 30.

AN APPEAL FOR SUPPORT OF THE NEWS LETTER

(1) From two nationally and internationally known mathematicians have come the following unsolicited appreciations:

Dartmouth College, Department of Mathematics

Hanover, N. H., December 4, 1928.

Mr. S. T. Sanders,

I have just received a copy of your Mathematics News Letter. I do not know who is responsible for sending me this copy but I am very glad indeed to have it and I want to congratulate you and your Association in Louisiana and Mississippi on your success in publishing such a journal. It is very gratifying to all of us who are interested in the further development of Mathematics in this country and the improvement of its teaching to note the energy and initiative which is being shown by your group.

Very sincerely yours,

J. W. YOUNG

Brown University

Providence, R. I., December 1, 1928

Dear Professor Sanders,

I have just received Mathematics News Letter Vol. 3, No. 3. Please send me Vol. 1, No. 1 to Vol. 3, No. 2 inclusive and later numbers as published. I enclose a check for $2. I will pay the balance on the presentation of the bill.

Very sincerely yours,

R. C. ARCHIBALD

When those who are accounted among our greatest mathematicians find praiseworthy values in the News Letter, editorial appeal for a more generous support of it by the rank and file of mathematical workers must at least merit a hearing.

(2) The News Letter is worth to the progressive mathematics teacher **more than its subscription price.**

(3) It is the only journal in America that undertakes to represent the **consolidated** interests of high school and college mathematics teachers.

(4) Some of the contributors to its pages are classed among **the best teachers and scholars of the country.**

(5) **Money to print the News Letter is sorely needed.**

PROBLEMS

Proposed by H. L. SMITH

1. Show that in any triangle

$$\frac{S^2}{K}\,\frac{\tan \tfrac{1}{2}(A+B)}{\tan \tfrac{1}{2}\tan \tfrac{1}{2} B} = \operatorname{ctn} \tfrac{1}{2} A \operatorname{ctn} \tfrac{1}{2}B \operatorname{ctn} \tfrac{1}{2}C.$$

where A, B are any two angles of the triangle, S is its semi-perimeter, and K is its area.

2. Show that in any triangle

$$\frac{S^2}{K} \leqq \frac{(1+\sqrt{1-e^2})^2}{e\sqrt{1-e^2}} < \frac{8}{e},$$

where s, K are as above, and e is a positive number, at most equal to the smallest of the sines of the angles of the triangle.

MEETING OF MATHEMATICS BODIES

The purpose of this notice is to call attention to the fact that during the latter part of March The La.-Miss. Section of the Mathematical Association of America, and the La.-Miss. Branch of the National Council of Teachers of Mathematics, will hold their regular annual joint meeting in the city of Lafayette, Louisiana. At the meeting of these organizations in Jackson, Miss., last year there was a good attendance of mathematics teachers from these two states, and there was a considerable amount of interest manifested. It is hoped that this interest will bring together at the Lafayette meeting an even better attendance of mathematics teachers from the two states. Mathematical study and discussion afford a splendid field for both the practical and theoretical, and it is the desire of the officers and members of these organizations that there shall be a more widespread interest in the subject throughout these two states.

W. C. ROATEN,
Chairman, La.-Miss. Branch, DeRidder, La.